First Mid-term Examination

Sub:- Digital Signal Processing.

Computer Science 8CS.

Q.1.

(a): Explain Continuous Time Processing of Discrete Time Signals?

Ans In this section the input and output are in discrete form whereas the intermediate signals are in continuous in nature as shown in fig. 1.10 which approximates the continuous time processing of discrete time signals.

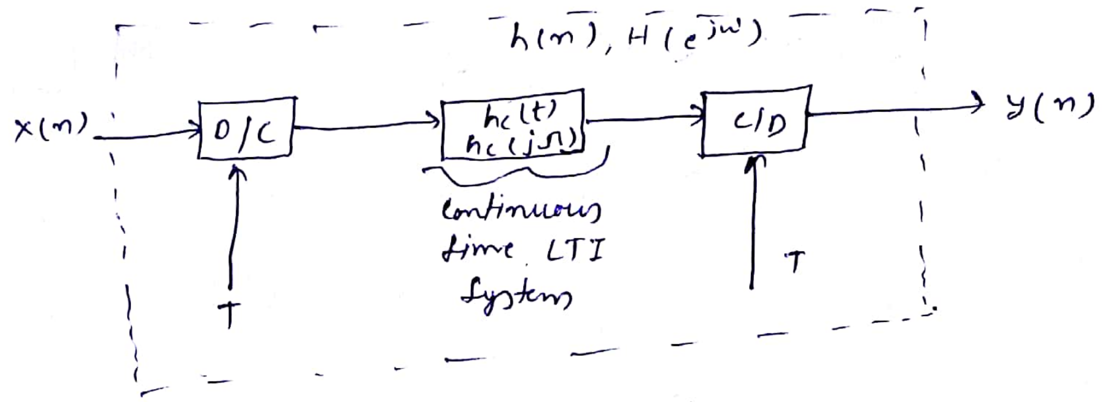


Fig:- Continuous time processing of discrete time signals.

The output of the D/C converter as discussed in the previous section, is given as

$$x_c(t) = \sum_{n=-\infty}^{\infty} x_n \frac{\sin[\pi(t-nT)/T]}{\pi(t-nT)/T} \quad \text{---} (1)$$

and the output of the continuous time LTI system is given as

$$y_c(t) = \sum_{n=-\infty}^{\infty} y_n \frac{\sin[\pi(t-nT)/T]}{\pi(t-nT)/T} \quad \text{---} (2)$$

In the fig. $x(n) = x_a(nT)$ and $y(n) = y_c(nT)$.

The freq. relationship for figure

$$x_c(j\Omega) = H_r(j\Omega) \cdot X(e^{j\Omega T}) \quad \text{---} (3)$$

Let us consider that the gain of the ideal reconstruction filter is T then

$$|H_r(j\Omega)| = T \quad ; \quad -\frac{\pi}{T} \le \Omega \le \frac{\pi}{T} \quad \text{--- (4)}$$

From eqn (3) and (4), we get

$$x_c(j\Omega) = T x(e^{j\Omega T}) \quad ; \quad -\frac{\pi}{T} \le \Omega \le \frac{\pi}{T} \quad \text{--- (5)}$$

and $$Y_c(j\Omega) = H_c(j\Omega)\, x_c(j\Omega) \quad ; \quad -\frac{\pi}{T} \le \Omega \le \frac{\pi}{T} \quad \text{--- (6)}$$

and $$Y(e^{j\omega}) = \frac{1}{T} Y_c\left(j\frac{\omega}{T}\right) \quad \text{[from eqn.]} \quad \text{--- (7).}$$

Substituting the eqn. (6) in eqn (7), we get.

$$Y(e^{j\omega}) = \frac{1}{T}\left[H_c\left(j\frac{\omega}{T}\right) x_c\left(j\frac{\omega}{T}\right)\right] \quad \text{--- (8)}$$

substituting the eqn. (4) in eqn (8), we get.

$$Y(e^{j\omega}) = \frac{1}{T}\left[H_c\left(j\frac{\omega}{T}\right) T x\left(e^{j\left(\frac{\omega}{T}\right)T}\right)\right]$$

$$Y_c(j\omega) = H_c\left(j\frac{\omega}{T}\right) x_c(j\omega)$$

or. $$\frac{Y_c(j\omega)}{X_c(j\omega)} = H_c\left(j\frac{\omega}{T}\right)$$

or. $$H(e^{j\omega}) = H_c\left(j\frac{\omega}{T}\right) \quad ; \quad -\pi \le \omega \le \pi \quad \text{--- (9).}$$

The eqn (9) denotes that the whole system behave as a discrete time system whose frequency response is defined in eqn. (10).

putting $\omega = \Omega T$ in eqn. (9), we get.

$$H(e^{j\Omega T}) = H_c\left(j\frac{\Omega T}{T}\right) ; \quad -\pi \le \Omega T \le \frac{\pi}{T}$$

or. $$H(e^{j\Omega}) = H_c(j\Omega) ; \quad -\frac{\pi}{T} \le \Omega \le \frac{\pi}{T}$$

or. $$H_c(j\Omega) = H(e^{j\Omega}) ; \quad -\frac{\pi}{T} \le \Omega \le \frac{\pi}{T} \quad \text{--- (}$$

eqⁿ (10) denotes that the whole response of the system in figure will be equal to a given $H(e^{j\omega})$ if the frequency response of the continuous time system is given by the eqⁿ. Since $X_c(j\Omega) = 0$ for $|\Omega| \geq \pi/T$, $H_c(j\Omega)$ may be chosen arbitarily above π/T, but arbitary choice is $H_c(j\Omega)=0$ for $|\Omega| \geq \pi/T$.

1.(b): A continuous signal is given as

$$x_c(t) = \sin(2\pi(100)t)$$

This signal was sampled with sampling period $T = \frac{1}{400}$ sec. to obtain discrete time signal $x(n)$. Determine the resulting signal $x(n)$.

Ans.

$$T = \frac{1}{400} \text{ sec.}$$

The discrete time signal is obtained from the continuous-time signal as

$$x(n) = x_c(nT) \quad \text{--- (1)}$$

$$x_c(t) = \sin(2\pi(100)t)$$

or.

$$x_c(nT) = \sin(2\pi((100)nT)$$

$$x(n) = x_c(nT) = \sin(2\pi(100)nT) \quad \text{-- (2)}$$

putting $T = \frac{1}{400}$, we get

$$x(n) = \sin\left[2\pi(100)n\frac{1}{400}\right]$$

$$= \sin\left[2\pi\left(\frac{n}{4}\right)\right]$$

$$= \sin\left(\frac{\pi n}{2}\right)$$

Q. (2).

(a). Explain Frequency Domain Representation of Down Sampling or Compressor.

Ans. As we know that $x(n) = x_c(nT)$ and from the previous discussion it is clear that the Discrete Time Fourier Transform of $x(n)$ is given as.

$$X(e^{jw}) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X_c\left[j\left(\frac{w}{T} - \frac{2\pi k}{T}\right)\right] \quad \text{--- (1)}$$

but $x_d(n) = x(nM) = x_c(nMT) = x_c(nT')$

where $T' = MT$.

$$X_d(e^{jw}) = \frac{1}{T'} \sum_{r=-\infty}^{\infty} X_c\left[j\left(\frac{w}{T'} - \frac{2\pi r}{T'}\right)\right]$$

so,

$$X_d(e^{jw}) = \frac{1}{MT} \sum_{r=-\infty}^{\infty} X_c\left(j\left(\frac{w}{MT} - \frac{2\pi r}{MT}\right)\right) \quad \text{--- (2)}$$

For obtaining the relation b/w eqn (1) and (2), the summation index r can be given as

$$r = i + kM \quad \text{--- (3)}$$

where k and i are integers such that $-\infty < kM < \infty$ and $0 \le i \le M-1$. clearly r is still and integer ranging from $-\infty$ to ∞. The eqn (2) can be written as.

$$X_d(e^{jw}) = \frac{1}{MT} \sum_{i=0}^{M-1} \sum_{k=-\infty}^{\infty} X_c\left[j\left(\frac{w}{MT} - \frac{2\pi(i+kM)}{MT}\right)\right]$$

$$= \frac{1}{M} \sum_{k=-\infty}^{M-1} \left[\frac{1}{T} \sum_{k=-\infty}^{\infty} X_c\left[j\left(\frac{w}{MT} - \frac{2\pi i}{MT} - \frac{2\pi kM}{MT}\right)\right]\right]$$

$$= \frac{1}{m}\sum_{i=0}^{m-1}\left[\frac{1}{T}\sum_{k=-\infty}^{\infty} X_c\left(j\left(\frac{\omega}{mT}-\frac{2\pi i}{mT}-\frac{2\pi k}{T}\right)\right)\right]$$

$$= \frac{1}{m}\sum_{i=0}^{m-1}\left[\frac{1}{T}\sum_{k=-\infty}^{\infty} X_c\left(j\left(\frac{\omega}{mT}-\frac{2\pi k}{T}-\frac{2\pi i}{mT}\right)\right)\right] \quad \text{--- (4)}$$

$$= \frac{1}{m}\sum_{i=0}^{m-1}\left[\frac{1}{T}\sum_{k=-\infty}^{\infty} X_c\left(j\left(\frac{\omega-2\pi i}{mT}-\frac{2\pi k}{T}\right)\right)\right] \quad \text{--- (5)}$$

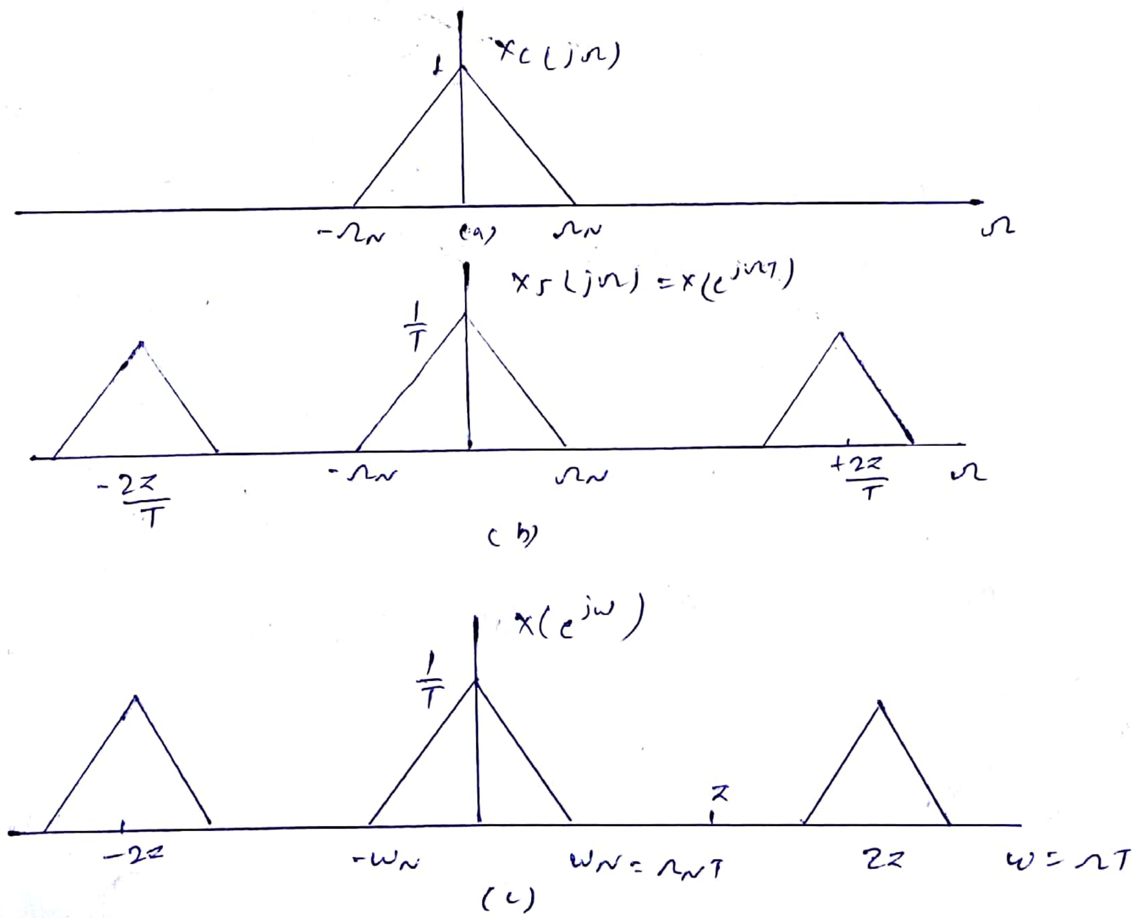
$$= \frac{1}{m}\sum_{i=0}^{m-1}\left[\frac{1}{T}\sum_{k=-\infty}^{\infty} X_c\left(j\left(\frac{(\omega-2\pi i)/m}{T}-\frac{2\pi k}{T}\right)\right)\right] \quad \text{--- (6)}$$

In eqn (6) it is clear that.

$$\frac{1}{T}\sum_{k=-\infty}^{\infty} X_c\left[j\left(\frac{(\omega-2\pi i)/m}{T}-\frac{2\pi k}{T}\right)\right] = X\left(e^{j(\omega-2\pi i)/m}\right) \quad \text{--- (7)}$$

eqn (6) and (7) we get.

$$X_d(e^{j\omega}) = \frac{1}{m}\sum_{i=0}^{m-1} X\left(e^{j\left(\frac{\omega}{m}-\frac{2\pi i}{m}\right)}\right) \quad \text{--- (8)}$$



Q.(2)(b): The continuous-time signal is given by

$$x_c(t) = \sin(20\pi t) + \cos(40\pi t)$$

This signal is sampled with a sampling period T to obtain the discrete-time signal.

$$x(n) = \sin\left(\frac{\pi n}{5}\right) + \cos\left(\frac{2\pi n}{5}\right)$$

(i) Determine the choice for T consistent with this information

(ii) Is your choice for T in part (i) unique? If so, explain why? If not specify another choice of T consistent with the information given.

Ans. (i) Let us consider that $T = \frac{1}{100}$ and it gives.

$$x(n) = x_c(nT)$$

$$= \sin\left(20\pi n \frac{1}{100}\right) + \cos\left(40\pi n \frac{1}{100}\right)$$

$$= \sin\left(\frac{\pi n}{5}\right) + \cos\left(\frac{2\pi n}{5}\right) \quad \text{---}(1)$$

The eqn (1) is identical to eqn given in problem

So $T = \frac{1}{100}$

(ii) No, our choice for T in part is not unique. because another choice is $T = \frac{11}{100}$

$$x(n) = x_c(nT)$$

$$= \sin\left(20\pi n \frac{11}{100}\right) + \cos\left(40\pi n \frac{11}{100}\right)$$

$$= \sin\left(\frac{11\pi n}{5}\right) + \cos\left(\frac{22\pi n}{5}\right)$$

$$= \sin\left(\frac{\pi n}{5}\right) + \cos\left(\frac{2\pi n}{5}\right)$$

So, another choice for T is $T = \frac{11}{100}$